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Cartan geometries and Lie algebroids

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Abstract

"A Cartan geometry is a Klein geometry with curvature": that is, given a Klein geometry as a homogeneous space G/H where G is a Lie group and H a closed Lie subgroup, a Cartan geometry is a smooth manifold M which locally is "like G/H".

A modern approach to Cartan geometry is given in a book by Sharpe, where the structure is given by a principal H-bundle over M and a "Cartan connection", a 1-form on M taking values in the Lie algebra of G (rather than H).

In this series of lectures I shall describe an alternative approach to Cartan geometry starting with, rather than a principal bundle, a fibre bundle with standard fibre G/H. The morphisms of this structure form a Lie groupoid with a distinguished Lie subgroupoid, and the geometry is given by a path connection. The corresponding infinitesimal structures are Lie algebroids and an infinitesimal connection.

An advantage of this approach is that the Lie algebroids obtained in this way can be identified with certain Lie algebroids of projectable vector fields on the fibre bundle. This gives a means of relating the present approach to those classical studies of projective and conformal geometry which used methods of tensor calculus. An extension of this method can also be used to study the more general projective geometry of sprays.