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Title: Invariant variational problems and cohomology

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Abstract: Globally hyperbolic spacetimes form probably the most important class of spacetimes. Low conjecture and the Legendrian Low conjecture formulated by Natário and Tod say that for many globally hyperbolic spacetimes X two events x, y in X are causally related if and only if the link of spheres S_x, S_y whose points are light rays passing through x and y is non-trivial in the contact manifold N of all light rays in X. This means that the causal relation between events can be reconstructed from the intersection of the light cones with a Cauchy surface of the spacetime.
We prove the Low and the Legendrian Low conjectures and show that similar statements are in fact true in almost all 4-dimensional globally hyperbolic spacetimes. This also answers the question on Arnold's problem list communicated by Penrose.
We also show that on many 4-manifolds there is a unique smooth structure underlying a globally hyperbolic Lorentz metric, thus global hyperbolicity imposes censorship on the possible smooth structures on a spacetime. For instance, every contractible smooth 4-manifold admitting a globally hyperbolic Lorentz metric is diffeomorphic to the standard R^4.