

Energy-momentum tensor and variational completion

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The idea of finding the energy-momentum tensor of a classical field theory as a kind of „Noether current” associated to the invariance of the corresponding matter Lagrangian to some (or even the entire) group of spacetime diffeomorphisms is already many decades old and it was, for sure, a subject of numerous debates.

The majority of the approaches start from the canonical, or Noether energy-momentum tensor - or from a covariant version of it – and try to „improve” it, by adding correction terms; this way, one finds, in the particular case of general relativity, the Hilbert energy-momentum tensor as a consequence. In this respect, a remarkable paper is the one by Gotay and Marsden (2001), which presents a geometric approach to the problem, on general fibered manifolds.

After a brief review of the problems surrounding energy-momentum tensors and of the Gotay-Marsden improvement procedure (with some clarifications by Forger and Romer, 2004), we redo the whole construction the other way around. Namely, we define energy-momentum tensors via a generalized Hilbert-type procedure and find out that they give the correct Noether currents as a consequence. This not only simplifies the computations and the proofs of the corresponding results, but also opens up new possibilities. One of these possibilities is the use of the technique of variational completions for finding the full expression of an energy-momentum tensor in the case when we know by some means a piece of it – and even finding a Lagrangian for the theory, in the case when this is not known.

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