18th International Summer School

on

Global Analysis and its Applications

EXTENDED ABSTRACT BOOK

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Levoča, Slovakia

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18th International Summer School on Global Analysis and its Applications
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Preface

The tradition of the Summer School on Global Analysis and its Applications was founded by prof. D. Krupka in 1996, in his birthplace, beautiful medieval town of Levoča, written in the UNESCO list (http://whc.unesco.org/en/list/620). Since 1996, the Summer School has been organized every year by prof. Krupka and his collaborators in several Slovak and Czech places, and has become a well-known scientific event all over the world.

Present Summer School has been again held in Levoča, 12-17 August, 2013. In past, the 11th Summer School was organized as Satellite Conference of the International Congress of Mathematicians, and last year, the 17th Summer School was included among the Satellite Meetings of the 6th European Congress of Mathematics.

The programme of this school is devoted to the Local and Global Inverse Problem of the Calculus of Variations. The main lectures are given by recognized specialists in the field. Moreover, to support the young participants in their scientific activity, the commented poster session and short presentations were organized.

Abstracts of main lecture series and short communications, contained in this book, summarize theoretical and applied aspects of the topics, considered at the school. They also provide the reader with basic information on research interest of participants and further orientation of their work in these fields.

The organizers would like to thank the Faculty of Humanities and Natural Sciences, University of Prešov, Slovakia, the University of Ostrava, Czech Republic, for their support to the Summer School. The organizers also appreciate the help of Ms. Jana Verešpejová during the Summer School.

Levoča, 16 August 2013 Ján Brajerčík

Chairman of the Organizing Committee

Programme

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CONTROLLED LAGRANGIANS AND THE INVERSE PROBLEM OF THE CALCULUS OF VARIATIONS

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Abstract

The inverse problem of calculus of variations, in its simplest setting, studies when a second order vector field is variational. The answer is given, locally, by the so-called Helmholtz conditions. A controlled vector field is a vector field on a manifold (called state space) that depends on parameters. Control theory studies how to assign these parameters as functions of time and/or state variables in order to accomplish the desired properties of dynamics. For this reason, these parameters are referred to as controls. An important special class of controlled vector fields are mechanical controlled vector fields. These are second order controlled vector fields that are Lagrangian when the controls are set to zero. Of course, specifying the controls generically leads to a non-Lagrangian vector field. It may be, however, interesting and desirable to select controls in such a way that the controlled dynamics is Lagrangian, too. This is the idea behind the method of controlled Lagrangians for stabilization of (relative) equilibria of mechanical systems. For the task to be accomplished, certain matching conditions should be satisfied. Since the controlled dynamics is Lagrangian, it is natural to conjecture that there is a connection between the Helmholtz and matching conditions. We will review the main concepts of the inverse problem of calculus of variations, matching stabilization technique, and elucidate the links between the Helmholtz and matching conditions.

MSC2010: 49N45, 70Q05, 70H03, 93D15
Keywords: inverse problem, calculus of variations, controlled Lagrangians, feedback stabilization.

THE INVERSE PROBLEM OF THE CALCULUS OF VARIATIONS: AN INTRODUCTION

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Abstract

The *inverse problem of the calculus of variations* is the problem of finding conditions, ensuring that a given system of (ordinary or partial) differential equations coincides with the system of Euler-Lagrange equations of an integral variational functional. Its origin, dated 1886, is connected with the names of Sonin and Helmholtz; a newer modified version of the inverse problem for systems of ordinary second order equations, using *variational integrating factors*, was presented by Douglas in 1941. Since then the problem, lying on the border of the calculus of variations, mathematical analysis of differential equations, differential geometry, and topology of manifolds was studied by many authors. However, in its generality it still belongs to mathematical problems that wait for a complete solution. The aim of this lecture series is to give an introduction to the local and global inverse problem.

First we consider the variationality problem for systems of ordinary second order differential equations. We derive the Helmholtz variationality conditions and find integrability conditions for the Douglas's problem.

The global inverse problem is then formulated within the global variational theory, extending the classical calculus of variations from Euclidean spaces to smooth manifolds. The problem is to find conditions when a system of equations on a manifold, which is locally variational, admits a *global* Lagrangian. We introduce underlying variational concepts in terms of differential forms, and study the theory of variational sequences, in which one arrow represents the Euler-Lagrange mapping of the calculus of variations. The sequence relates

properties of the Euler-Lagrange mapping with the De Rham cohomology of the underlying manifold.

In these lectures we do not consider the inverse problem for vector fields on tangent bundles (sprays), which is related with the Douglas's problem.

Contents

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MSC2010: 49N45, 58A20, 58E30, 49Q99
Keywords: inverse problem, jet, variational principle, fibred manifold.

HOMOGENEOUS VARIATIONAL PROBLEMS: A MINICOURSE

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Abstract

A Finsler geometry may be understood as a homogeneous variational problem, where the Finsler function is the Lagrangian. The extremals in Finsler geometry are curves, but in more general variational problems we might consider extremal submanifolds of dimension m. In this minicourse we discuss these problems from a geometric point of view.

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MSC2010: 35A15, 58A10, 58A20

Keywords: calculus of variations, parametric problems.

THE GEOMETRY OF VARIATIONAL PRINCIPLES ON GRASSMANN FIBRATIONS

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Abstract

The Grassmann fibrations are regarded in this lecture as basic geometric underlying structures for variational problems for submanifolds. The corresponding variational objects (Lagrangian, Euler-Lagrange form, Helmholtz form, Noether current) appear in this context to be rather classes than differential forms as such. The notion of a Lepage form on Grassmann fibration is introduced. We consider the variational functionals associated with Lepage forms for higher-order immersed curves and first-order immersed submanifolds and discuss the inverse variational problem in terms of Helmholtz conditions. The correspondence with homogeneous variational problems on slit tangent bundles will be discussed.

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Keywords: Lagrangian, Euler-Lagrange form, Lepage form, submanifold, jet, contact element, invariance, Noether current

NATURAL LAGRANGIANS OF FRAMES AND OF COFRAMES

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Abstract

Let FX and F^*X denote the frame and the coframe bundle, respectively, over an n-dimensional manifold X. By *natural Lagrangian* we mean Lagrangian invariant with respect to all diffeomorphisms of X. Variational principles defined by natural Lagrangians on frame and coframe bundles are well known in several approaches to gravitation, and general relativity. One of the tasks of differential geometry is to describe the structure of all natural Lagrangians of the corresponding order r.

Let L^r_n be r-th differential group of $\mathbb{R}^n$ (n-tuples of real numbers). Let P and Q be two manifolds with left actions of L^r_n. A smooth mapping $F : P \to Q$ is called a *differential invariant* if it is L^r_n-equivariant, i.e., $F(g \cdot p) = g \cdot F(p)$ for all $g \in L^r_n$ and $p \in P$. A characteristic property of a natural Lagrangian is that it is, in fact, a differential invariant. Effective tool to find these differential invariants is the *orbit reduction method*, first time used in D. Krupka, *Local invariants of a linear connection*, In: Diff. Geometry, Budapest, 1979, Colloq. Math. Soc. J. Bolyai **31**, North Holland, Amsterdam, 1982. The orbit reduction method uses the fact that L^r_n can be represented as a *semi-direct product* of L^1_n and the kernel of canonical jet projection of L^r_n onto L^1_n, denoted by K^r_n, i.e., $L^r_n = L^1_n \times_s K^r_n$.

The frame and coframe bundles of X can be interpreted as fibre bundles associated with the principal bundle FX and type fibre L^1_n; left actions of the structure group L^1_n of FX on the type fibre are called *frame* and *coframe actions*, respectively. Then, by prolongation theory, r-jet prolongations J^rFX and J^rF^*X of frame and coframe bundles, respectively, have the structure of fibre bundles, associated with the principal bundle $F^{r+1}X$ of the frames of order

$r+1$, and with type fibre $T^r_n L^1_n$ (the manifold of r-jets with source $0 \in \mathbb{R}^n$ and target in L^1_n). The actions of L^{r+1}_n on the type fibre are induced by *prolongation formula* from frame and coframe actions.

General result on the structure of natural Lagrangians says that, given L^1_n-manifold Q, there is a one-to-one correspondence between natural Lagrangians on $J^r F_Q X$ and differential invariants $\mathcal{I} : T^r_n Q \to \widetilde{\mathbb{R}}$, where $\widetilde{\mathbb{R}}$ is the real line endowed with the trivial action of L^1_n (see, e.g., D. Krupka, *Natural variational principles*, In: Symmetries and Perturbation Theory, Proc. of the Internat. Conf., Otranto, Italy, 2007, World Scientific, 2008, pp. 116–123). Thus, to obtain all natural Lagrangians of order r on frame and coframe bundles it is sufficient to describe all differential invariants $\mathcal{I} : T^r_n L^1_n \to \widetilde{\mathbb{R}}$.

During calculation process of differential invariants $\mathcal{I}$ we use Young decomposition of tensors of the corresponding type. Calculation difficulty increases with the increasing of differential invariants order. There is also difference in calculation difficulty between differential invariants of frames and coframes of the same order. Because of duality of the frame and the coframe actions (see D. Q. Chao and D. Krupka, *3rd order differential invariants of coframes*, Math. Slovaca, **49** (1999), 563–576), it could be useful to obtain differential invariants of coframes first and then to transpose them into differential invariants of frames.

To finalize the construction of natural Lagrangians of frames and of coframes we introduce the concept of *canonical odd n-form on FX, and on F^*X*, respectively. In the case of frames the situation is as follows. Any chart (U, φ), $\varphi = (x^i)$, on X, induces the *fibred* chart (V, ψ), $\psi = (x^i, x^i_j)$, on FX. By setting $x^i_j y^j_k = \delta^i_k$ we define another coordinates y^j_k on FX. With the chart (V, ψ) we associate the object

$$\tilde{\omega}_{(V,\psi)} = |\det y^i_j| \cdot \tilde{\varphi} \otimes dx^1 \wedge dx^2 \wedge \ldots \wedge dx^n, \tag{1}$$

where $\tilde{\varphi}$ is a *field of odd scalars* on X, associated with (U, φ) (see D. Krupka, *Natural Lagrangian structures*, In: Banach Center Publications, **12**, Polish Scientific Publishers, Warszawa, 1984, pp. 185–210). It is easily seen that (1) represents a globally defined odd base form on FX; we denote this form by $\tilde{\omega}$, and call it the *canonical odd n-form on FX*. The *canonical odd n-form on F^*X*

can be obtained analogously. For the properties of these forms see J. Brajerčík, *Second order differential invariants of linear frames*, Balkan J. Geom. Appl., **15** (2010), no. 2, 14–25, and J. Brajerčík and M. Demko, *Second order natural Lagrangians on coframe bundles*, Miskolc Math. Notes, to appear, respectively.

Our main results for frames are described in the following assertions.

Theorem 1 *Every r-th order natural Lagrangian λ on frame bundles is of the form*

$$\lambda = \mathcal{L}\tilde{\omega},$$

where $\mathcal{L}$ is any function of order r, invariant with respect to corresponding lifts of all diffeomorhisms of X, and $\tilde{\omega}$ is canonical odd n-form of FX.

For illustration we give an explicit expression of the functions $\mathcal{L}$ for $r = 2$. Let (V^2, ψ^2), $\psi^2 = (x^i, x^i_j, x^i_{j,k}, x^i_{j,kl})$ be the fibred chart on J^2FX, *associated* with (V, ψ).

Theorem 2 *Any function $\mathcal{L}$ on J^2FX, invariant with respect to corresponding lifts of all diffeomorhisms of X, can be locally written as a differentiable function of the functions $\mathcal{L}^i_{j,k}$, $\mathcal{L}^i_{jk,l}$ which are given by*

$$\begin{aligned}
\mathcal{L}^i_{j,k} &= y^i_t x^s_{\tilde{k}} x^t_{\tilde{j},s}, \\
\mathcal{L}^i_{jk,l} &= y^i_t (2x^s_l x^m_{\tilde{k}} x^t_{\tilde{j},ms} + y^p_q x^s_l x^t_{p,s} x^m_{\tilde{j}} x^q_{\tilde{k},m} + x^s_{\tilde{j}} x^t_{l,m} x^m_{\tilde{k},s} + \tfrac{3}{2} x^s_l x^t_{\tilde{j},m} x^m_{\tilde{k},s} \\
&\quad + \tfrac{1}{2} x^s_{\tilde{k}} x^t_{\tilde{j},m} x^m_{l,s} + \tfrac{1}{2} y^p_q x^s_l x^t_{p,m} x^m_{\tilde{j}} x^q_{\tilde{k},s} + \tfrac{1}{2} y^p_q x^s_{\tilde{k}} x^m_{\tilde{j}} x^t_{p,s} x^q_{l,m}),
\end{aligned}$$

where writing a tilde over two indices means antisymmetrization in these indices.

Second order natural Lagrangians on coframe bundles have been obtained by an analogous way.

MSC2010: 53A55, 58A10, 58A20
Keywords: natural Lagrangian, frame, coframe, differential invariant.

GLUING FRIEDMAN COSMOLOGICAL MODELS IN DIFFERENTIAL SPACES THEORY

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Abstract

Friedman cosmological models are fundamental in classical modern cosmology. In cyclic models one has to glue universes, which usually produces a "singularity". Therefore a formal description must be subtle. However it occurs that when the description is done in theory of differential spaces, the problem of singularities is easier to cope with. Differential spaces (in a sense of Sikorski) are therefore a very useful generalizations of the concept of a classical smooth manifold. A special subcategory of differential spaces are so called differential spaces generated by some (in our case finite) family of functions. These generators can play the role of a nice tool in a special technique of gluing two differential spaces (named of course "generator gluing method"). Classically there emerges a problem with smoothness of functions and one has to smooth the obtained edge or shift. In the category of differential spaces functions must be only continuous. Therefore no "smoothing" procedure is needed even if some kind of a "singular" point emerges. The proposed technique allows to glue functions, vector fields, differential forms and all geometric objects. The whole technique is not fully developed, but first results indicate that it is worth to study this concept more thoroughly. From mathematical point of view this formalism is an interesting part of global analysis.

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MSC2010: 58A40, 83C75, 83F05
Keywords: Differential spaces, Friedman cosmological model, singularity, cyclic cosmology.

PRESYMPLECTIC CURRENT AND THE INVERSE PROBLEM OF THE CALCULUS OF VARIATIONS

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Abstract

The inverse problem of the calculus of variations asks whether a given system of partial differential equations (PDEs) admits a variational formulation. We show that the existence of a pre-symplectic form in the variational bicomplex, when horizontally closed on solutions, allows us to construct a variational formulation for a subsystem of the given PDE. No constraints on the differential order or number of dependent or independent variables are assumed. Uniqueness of the variational formulation is also discussed but is inconclusive.

The PDE can be given in any form. Hence, the inverse problem of interest is of the hard, multiplier kind. Our result can be considered as a PDE analog of the older and stronger result of Henneaux [1] for systems of ordinary differential equations (ODEs). Henneaux used the following crucial concepts, that were available at the time, for the geometric formulation of ODEs: (a) ODE as a vector field, (b) definition of symplectic form from Lagrangian, (c) conservation of symplectic form under Lie flow of the ODE, (d) non-degeneracy of ODE and symplectic form. Unfortunately, his result proved difficult to generalize to PDEs [2] because the analogs of the above concepts for PDEs were not well known.

Gradually, the right concepts became available and better known in the course of the development of the literature on the geometric formulation of PDEs in terms of *jet bundles* and the associated *variational bicomplex*, as well as the *local symplectic structure* of field theories. Finally, a short remark of Hydon [3] and Bridges, Hydon and Lawson [4] contributed the crucial idea for recovering a Lagrangian from using the following PDE concepts in place of the

corresponding ones for ODEs: (a) PDE as a jet bundle submanifold, (b) definition of local pre-symplectic current from Lagrangian, (c) on-shell conservation condition via the variational bicomplex. We have expanded on this remark, placed it in the appropriate geometric context and related it to the older work of Henneaux.

Unfortunately, the right analog the non-degeneracy conditions for PDEs are not currently apparent. So the uniqueness of the obtained Lagrangian and the equivalence of its Euler-Lagrange equations to the original PDE system cannot be stated conclusively. Still we pose and sharpen these questions with the help of a certain pre-order on Lagrangians, defined in terms of their Euler-Lagrange equations and pre-symplectic currents.

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MSC2010: 58E30, 35R30, 53D05
Keywords: inverse problem, calculus of variations, presymplectic current.

SUPERSTABILITY OF AN EXPONENTIAL EQUATION IN C^*-ALGEBRAS

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Abstract

The aim of this paper is to prove the superstability of the following functional equations

$$f\Big(\frac{x+y}{m}\Big)^m \;=\; g(x)h(y),$$

where $f, g, h : V^2 \to A$ are unknown mappings and m is a fixed positive integer. Here V is a vector space, and A is a unital normed algebra.

Furthermore, we prove the superstability of the following generalized Pexider exponential equation

$$f\Big(\frac{x+y}{r}\Big)^r = g(x)h(y),$$

where $f, g, h : V^2 \to I(A) \cap A^+$ are unknown mappings and r is a fixed nonzero rational number. Here V is a vector space, $I(A)$ is the set of all invertible elements in a commutative unital C^*-algebra A and A^+ is the positive cone of A.

Theorem A. Let $\varphi : V \times V \to \mathbb{R}_+ \cup \{0\}$ be a function. Assume that $\varphi(x, y)$ is bounded as a function of y for each $x \in V$, and that $f, g, h : V \to A$ satisfy the inequality

$$\left\| f\left(\frac{x+y}{m}\right)^m - g(x)h(y) \right\| \le \varphi(x, y)$$

for all $x, y \in V$ and $g(0) = I$. If there exists a sequence $\{y_n\}$ in V such that

$$\|h(y_n)^{-1}\| \to 0$$

as $n \to \infty$, then g satisfies

$$f(x+y) = f(x)f(y). \tag{E}$$

Theorem B. Let $\varphi : V \times V \to \mathbb{R}_+ \cup \{0\}$ be a function. Assume that $\varphi(x,y)$ is bounded as a function of y for each $x \in V$, and that $f, g, h : V \to I(A) \cap A^+$ satisfy the inequality

$$\left\| f\left(\frac{x+y}{r}\right)^r - g(x)h(y) \right\| \leq \varphi(x,y)$$

for all $x, y \in V$ and $g(0) = I$. If there exists a sequence $\{y_n\}$ in V such that

$$\|h(y_n)^{-1}\| \to 0$$

as $n \to \infty$, then g satisfies (E).

MSC2010: 39B52, 33B10, 65F10, 11D61, 46L05
Keywords: Hyers-Ulam stability, superstability, C^*-algebra, generalized Pexider exponential equation.

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VARIATIONAL SEQUENCES

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Abstract

The variational sequence theory on fibred manifolds with 1-dimensional base (fibred mechanics) are studied.

Let Y be a fibred manifold over 1-*dimensional base* X with *projection* $\pi : Y \to X$, and denote $m = \dim Y - 1$. Let $r \geq 0$. We denote by J^rY the *r-jet prolongation* of Y, and by $\pi^r : J^rY \to X$ and $\pi^{r,s} : J^rY \to J^sY$, $0 \leq s \leq r$, the canonical jet projections. If $r = 0$, we set $J^0Y = Y$. For any open subset $W \subset Y$, let $W^r = (\pi^{r,0})^{-1}(W) \subset J^rY$. An element of J^rY, denoted by $J^r_x\gamma$, is the r-jet of a section γ of Y with *source* $x \in X$ and *target* $\gamma(x) \in Y$.

We consider J^rY with standard geometric structures. Recall that every fibred chart (V, ψ), $\psi = (t, q^\sigma)$, $1 \leq \sigma \leq m$, on Y induces the chart (U, φ) on X, with $U = \pi(V)$, and the associated fibred chart (V^r, ψ^r) on J^rY, where $V^r = (\pi^{r,0})^{-1}(V)$, and $\psi^r = (t, q^\sigma, q^\sigma_1, q^\sigma_2, \dots, q^\sigma_r)$ is the collection of coordinates on V^r, defined by $q^\sigma_l(J^r_x\gamma) = D^l(q^\sigma\gamma\varphi^{-1})(\varphi(x))$. The associated charts (V^r, ψ^r) define a smooth structure of J^rY; the dimension of J^rY is given by $\dim J^rY = 1 + m(r + 1)$.

We denote by Ω^r_0W the ring of differentiable functions, defined on W^r, and by Ω^r_kW the Ω^r_0W-module of differentiable k-forms on W^r. The exterior algebra of forms on W^r is denoted by Ω^rW. Recall that the chart formulas $hf = f \circ \pi^{r+1,r}$, $h\,dt = dt$ and $h\,dq^\sigma_k = q^\sigma_{k+1}$ define a (global) homomorphism of exterior algebras $h : \Omega^rW \to \Omega^{r+1}W$, called the *horizontalisation*. Note that for any function $f : W^r \to R$, $h\,df = \frac{df}{dt}$, where $\frac{df}{dt} = \frac{\partial f}{\partial t} + \sum_{l=0}^{r} \frac{\partial f}{\partial y^\sigma_l} y^\sigma_{l+1}$ is the *formal* or *total derivative* of f.

A differential 1-form $\rho \in \Omega^r_1 W$, satisfying condition $h\rho = 0$, is said to be *contact*. Note that, evidently, every k-form $\rho \in \Omega^r_k W$, with $k \geq 2$, satisfies the condition $h\rho = 0$ trivially. Locally, every contact one form can be describe as

$$\rho = \sum_{l=0}^{r-1} B^l_\sigma \omega^\sigma_l$$

where

$$\omega^\sigma_l = dy^\sigma_l - y^\sigma_{l+1} dt.$$

Using the last expression, we can change the *standard basis* of 1-forms $dt, dq^\sigma_l, dq^\sigma_r$, $0 \leq l \leq r-1$ to the *contact basis* of 1-forms dt, ω^σ_l, dq^σ_r, $0 \leq l \leq r-1$, if it is suitable.

For any k-form $\rho \in \Omega^r_k W$ the pull-back $(\pi^{r+1,r})^* \rho$ has a unique decomposition $(\pi^{r+1,r})^* \rho = p_{k-1}\rho + p_k \rho$, called *canonical decomposition* of ρ. The form $p_{k-1}\rho$ (resp. $p_k\rho$) is called the $(k-1)$-*contact* (resp. k-*contact*) *component* of ρ. In particular, if $k = 1$, we denote $p_0\rho = h\rho$ the *horizontal* and $p_1\rho = p\rho$ the *contact component* of ρ, and write $(\pi^{r+1,r})^* \rho = h\rho + p\rho$,

A form $\rho \in \Omega^r_k W$ is said to be *k-contact*, or *completely contact*, if it is equal, up to the pull-back, to its k-contact component, that is, $(\pi^{r+1,r})^* \rho = p_k \rho$. The module of completely contact k-forms will be denoted by ${}^{(c)}\Omega^r_k W$.

We say that a k-form $\rho \in \Omega^r_k W$ is *contact*, if for every point from Y there exist a fibred chart (V, ψ), $\psi = (t, q^\sigma)$ and a completely contact $(k-1)$-form $\eta \in {}^{(c)}\Omega^r_{k-1} V$ such that $p_k(\rho - d\eta) = 0$. It is equivalent with assertion that ρ has a local expression $\rho = \mu + d\eta$ for some completely contact k-form μ and completely contact $(k-1)$-form η. Contact k-forms constitute an Abelian subgroup, denoted by $\Theta^r_k W$, of the Abelian group of k-forms $\Omega^r_k W$.

The subgroups $\Theta^r_k W$ of contact k-forms together with the exterior derivative operator d define a sequence

$$0 \to \Theta^r_1 W \to \Theta^r_2 W \to \ldots \to \Theta^r_M W \to 0, \tag{1}$$

where $M = mr + 1$. Sequence (1) is a subsequence of the De Rham sequence

$$0 \to \mathbb{R} \to \Omega^r_0 W \to \Omega^r_1 W \to \ldots \to \Omega^r_M W \to \Omega^r_{M+1} W \to \ldots \to \Omega^r_N W \to 0, \tag{2}$$

called the *contact subsequence*; the morphisms in (2) denote the exterior derivative operator and $N = \dim J^r Y = m(r+1) + 1$.

The quotient sequence

$$\begin{aligned} & 0 \to \mathbb{R} \to \Omega_0^r W \to \Omega_1^r W/\Theta_1^r W \to \dots \\ & \quad \to \Omega_M^r W/\Theta_M^r W \to \Omega_{M+1}^r W \to \dots \to \Omega_N^r W \to 0, \end{aligned}$$

with quotient mappings $E : \Omega_k^r W/\Theta_k^r W \to \Omega_{k+1}^r W/\Theta_{k+1}^r W$, defined on classes of forms by $E([\rho]) = [d\rho]$, , is called the *variational sequence of order* r *on* $J^r Y$.

Let us mention the quotient mapping $E : \Omega_2^r W/\Theta_2^r W \to \Omega_3^r W/\Theta_3^r W$ in more detail. Let ε be a differential 2-form given in fibred coordinates by $\varepsilon = \varepsilon_\sigma \omega^\sigma \wedge dt$, where $\varepsilon_\sigma = \varepsilon_\sigma(t, q^\sigma, q_1^\sigma, q_2^\sigma)$ (so called *source form*). The class of ρ is an element of $\Omega_2^r W/\Theta_2^r W$. Applying the definition of the quotient mapping we obtain a class $[d\rho]$, identified with 3-form on W^4

$$E(\varepsilon) = (H_{\sigma\nu}(\varepsilon)\omega^\nu + H^1_{\sigma\nu}(\varepsilon)\omega_1^\nu + H^2_{\sigma\nu}(\varepsilon)\omega_2^\nu) \wedge \omega^\sigma \wedge dt, \tag{3}$$

where

$$\begin{aligned} H^2_{\sigma\nu}(\varepsilon) &= \frac{\partial \varepsilon_\sigma}{\partial q_2^\nu} - \frac{\partial \varepsilon_\nu}{\partial q_2^\sigma}, \\ H^1_{\sigma\nu}(\varepsilon) &= \frac{\partial \varepsilon_\sigma}{\partial q_1^\nu} + \frac{\partial \varepsilon_\nu}{\partial q_1^\sigma} - 2\frac{d}{dt}\frac{\partial \varepsilon_\nu}{\partial q_2^\sigma}, \\ H^0_{\sigma\nu}(\varepsilon) &= \frac{\partial \varepsilon_\sigma}{\partial q^\nu} - \frac{\partial \varepsilon_\nu}{\partial q^\sigma} + \frac{d}{dt}\frac{\partial \varepsilon_\nu}{\partial q_1^\sigma} - \frac{d^2}{dt^2}\frac{\partial \varepsilon_\nu}{\partial q_2^\sigma} \end{aligned} \tag{4}$$

are the well-known *Helmholtz expressions.* Moreover, the form $E(\varepsilon)$ is equal to zero form if and only if the form ε is variational, i.e its coefficients ε_σ are the Euler-Lagrange expressions for some Lagrangian λ.

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Keywords: variational sequence, contact form, fibred mechanics, Lagrangian, Euler-Lagrange equations, Helmholtz variationality conditions.

POINCARÉ INVARIANCE OF THE HELMHOLTZ MORPHISM

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Abstract

We study the Helmholtz morphism in the variational sequence and find Helmholtz forms invariant with respect to the Poincaré group.

We shall use the framework of the theory of variational sequences on fibred manifolds. The variational sequence is a quotient sequence of the de Rham sequence, such that one of the morphisms is the *Euler-Lagrange morphism* $\mathcal{E}_1\colon \lambda \to E_\lambda$, assigning to a Lagrangian, i.e. one-form $\lambda = L\,\mathrm{d}t$, its Euler-Lagrange form, i.e. two-form $E_\lambda = E_\sigma(L)\,\mathrm{d}q^\sigma \wedge \mathrm{d}t$, where $E_\sigma(L)$ are the Euler-Lagrange expressions

$$E_\sigma(L) = \frac{\partial L}{\partial q^\sigma} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot q^\sigma}\,.$$

The next morphism $\mathcal{E}_2\colon E \to H_E$, called the *Helmholtz morphism*, assigns to a two-form $E = E_\sigma\,\mathrm{d}q^\sigma \wedge \mathrm{d}t$ a three-form H_E

$$\begin{aligned} H_E = {} & \frac{1}{2}\left(\frac{\partial E_\sigma}{\partial q^\nu} - \frac{\partial E_\nu}{\partial q^\sigma} - \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial E_\sigma}{\partial \dot q^\nu} - \frac{\partial E_\nu}{\partial \dot q^\sigma}\right)\right)\omega^\nu \wedge \omega^\sigma \wedge \mathrm{d}t \\ & + \frac{1}{2}\left(\frac{\partial E_\sigma}{\partial \dot q^\nu} + \frac{\partial E_\nu}{\partial \dot q^\sigma} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial E_\sigma}{\partial \ddot q^\nu} + \frac{\partial E_\nu}{\partial \ddot q^\sigma}\right)\right)\dot\omega^\nu \wedge \omega^\sigma \wedge \mathrm{d}t \\ & + \frac{1}{2}\left(\frac{\partial E_\sigma}{\partial \ddot q^\nu} - \frac{\partial E_\nu}{\partial \ddot q^\sigma}\right)\ddot\omega^\nu \wedge \omega^\sigma \wedge \mathrm{d}t. \end{aligned}$$

called *Helmholtz form.*

Classes in the variational sequence can be represented by differential forms. We shall use the representation by so-called *source forms*, $(q-1)$-contact q-forms belonging to the ideal generated by contact forms. We shall be interested in Helmholtz-like forms (the source forms representing the classes of local

3-forms) of order 3 (in particular, they correspond to second order ordinary differential equations). In coordinates,

$$H = H^0_{\sigma\nu}\,\omega^\nu \wedge \omega^\sigma \wedge \mathrm{d}t + H^1_{\sigma\nu}\,\dot\omega^\nu \wedge \omega^\sigma \wedge \mathrm{d}t + H^2_{\sigma\nu}\,\ddot\omega^\nu \wedge \omega^\sigma \wedge \mathrm{d}t$$

where $H^0_{\sigma\nu} = -H^0_{\nu\sigma}$, $H^1_{\sigma\nu} = H^1_{\nu\sigma}$, $H^2_{\sigma\nu} = -H^2_{\nu\sigma}$.

The Poincaré group on $\mathbb{R}^4$ is the 10-parametric transformation group, generated by the vector fields

$$\frac{\partial}{\partial q^0}, \quad \frac{\partial}{\partial q^1}, \quad \frac{\partial}{\partial q^2}, \quad \frac{\partial}{\partial q^3}$$

for the space-time translations, and

$$q^3\frac{\partial}{\partial q^2} - q^2\frac{\partial}{\partial q^3}, \quad q^1\frac{\partial}{\partial q^3} - q^3\frac{\partial}{\partial q^1}, \quad q^2\frac{\partial}{\partial q^1} - q^1\frac{\partial}{\partial q^2},$$
$$q^3\frac{\partial}{\partial q^0} + q^0\frac{\partial}{\partial q^3}, \quad q^1\frac{\partial}{\partial q^0} + q^0\frac{\partial}{\partial q^1}, \quad q^2\frac{\partial}{\partial q^0} + q^0\frac{\partial}{\partial q^2}$$

for the space-time rotations.

The problem is to find Helmholtz-like form H invariant with respect to the Poincaré group. Substituing the generators of the Poincaré group into symmetry conditions we obtain the following equations for the components of H:

$$H^0_{\sigma\rho}\frac{\partial\xi^\rho}{\partial q^\nu} + H^0_{\rho\nu}\frac{\partial\xi^\rho}{\partial q^\sigma} + \frac{\partial H^0_{\sigma\nu}}{\partial\dot q^\rho}\xi^\rho_1 + \frac{\partial H^0_{\sigma\nu}}{\partial\ddot q^\rho}\xi^\rho_2 + \frac{\partial H^0_{\sigma\nu}}{\partial\dddot q^\rho}\xi^\rho_3 = 0$$
$$H^1_{\rho\nu}\frac{\partial\xi^\rho}{\partial q^\sigma} + \frac{\partial H^1_{\sigma\nu}}{\partial\dot q^\rho}\xi^\rho_1 + \frac{\partial H^1_{\sigma\nu}}{\partial\ddot q^\rho}\xi^\rho_2 + \frac{\partial H^1_{\sigma\nu}}{\partial\dddot q^\rho}\xi^\rho_3 = 0$$
$$H^2_{\rho\nu}\frac{\partial\xi^\rho}{\partial q^\sigma} + \frac{\partial H^2_{\sigma\nu}}{\partial\dot q^\rho}\xi^\rho_1 + \frac{\partial H^2_{\sigma\nu}}{\partial\ddot q^\rho}\xi^\rho_2 + \frac{\partial H^2_{\sigma\nu}}{\partial\dddot q^\rho}\xi^\rho_3 = 0$$

and we solve them.

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MATRIX NORMED SPACES AND FUNCTIONAL EQUATIONS

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Abstract

In this talk, we prove the Hyers-Ulam stability of the Cauchy additive functional equation and the Cauchy additive functional inequality in matrix normed modules over a C^*-algebra.

Let E, F be vector spaces. For a given mapping $h : E \to F$ and a given positive integer n, define $h_n : M_n(E) \to M_n(F)$ by $h_n([x_{ij}]) = [h(x_{ij})]$ for all $[x_{ij}] \in M_n(E)$.

Throughout this paper, assume that A is a unital C^*-algebra with unitary group $U(A)$. Let $(X, \{\|\cdot\|_n\})$ be a matrix normed module over A and $(Y, \{\|\cdot\|_n\})$ a matrix Banach module over A.

For a mapping $f : X \to Y$, define $D_u f_n : M_n(X^2) \to M_n(Y)$ by

$$D_u f_n([x_{ij}], [y_{ij}]) := f_n(u[x_{ij} + y_{ij}]) - u f_n([x_{ij}]) - u f_n([y_{ij}])$$

for all $u \in U(A)$ and all $x = [x_{ij}], y = [y_{ij}] \in M_n(X)$.

Theorem A. Let $f : X \to Y$ be a mapping and let $\phi : X^2 \to [0, \infty)$ be a function such that

$$\Phi(a, b) := \frac{1}{2} \sum_{l=0}^{\infty} \frac{1}{2^l} \phi(2^l a, 2^l b) < +\infty,$$

$$\|D_u f_n([x_{ij}], [y_{ij}])\|_n \le \sum_{i,j=1}^{n} \phi(x_{ij}, y_{ij})$$

for all $a, b \in X$, $u \in U(A)$ and all $x = [x_{ij}], y = [y_{ij}] \in M_n(X)$. Then there exists a unique A-linear mapping $L : X \to Y$ such that

$$\|f_n([x_{ij}]) - L_n([x_{ij}])\|_n \le \sum_{i,j=1}^{n} \Phi(x_{ij}, x_{ij})$$

for all $x = [x_{ij}] \in M_n(n)$.

Theorem B. Let $f : X \to Y$ be a mapping and let $\phi : X^3 \to [0, \infty)$ be a function such that

$$\Phi(a, b, c) := \frac{1}{2}\sum_{l=0}^{\infty} \frac{1}{2^l}\phi(2^l a, 2^l b, 2^l c) < +\infty,$$

$$\begin{aligned} \|uf_n([x_{ij}]) + uf_n([y_{ij}]) + f_n(u[z_{ij}])\|_n &\le \|f_n([x_{ij}] + [y_{ij}] + [z_{ij}])\|_n \\ &+ \sum_{i,j=1}^{n} \phi(x_{ij}, y_{ij}, z_{ij}) \end{aligned}$$

for all $a, b, c \in X$, $u \in U(A)$ and all $x = [x_{ij}], y = [y_{ij}], z = [z_{ij}] \in M_n(X)$. Then there exists a unique A-linear mapping $L : X \to Y$ such that

$$\|f_n([x_{ij}]) - L_n([x_{ij}])\|_n \le \sum_{i,j=1}^{n} \Phi(x_{ij}, x_{ij}, -2x_{ij})$$

for all $x = [x_{ij}] \in M_n(X)$.

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Keywords: matrix normed module over a C^*-algebra; Cauchy additive functional inequality; Hyers-Ulam stability; Cauchy additive functional equation.

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MATHEMATICAL DESCRIPTION OF THE ADVANCED GEOMETRIC MUSCLE MODEL

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Abstract

Typical representatives of the artificial muscles are pneumatic artificial muscles (PAMs) which have a very good power-to-weight ratio and they are suitable for use as manipulators actuator. Due to their highly non-linear characteristics there are problems with control of such actuator and it is necessary to have suitable dynamic model of these muscles. One of the simple way to obtain it is mathematical describing considering muscle geometric properties. The advanced geometric muscle model in contrast to the simple model assumes that not only muscle length h changes when inflated or deflated through valve diameter d_1, but also muscle diameter d_2 changes. Geometry constants assuming non expansion braiding fibers are the half nylon thread length L and number N wrapped of single fibers (Fig. 1).

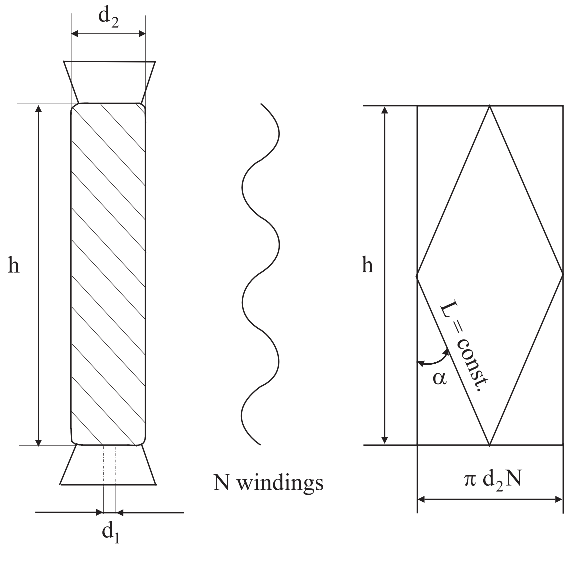

Fig.1 Geometry parameters of the PAM

The artificial muscle is modeled as an elliptic cylinder with non-zero thickness. Relation of the muscle cylinder volume V as a function of the muscle length h has the following form:

$$V = \frac{h \cdot [3d_1^2\pi^2N^2 + 4d_1\pi \cdot N\sqrt{4L^2 - h^2} + 8(4L^2 - h^2)]}{60\pi \cdot N^2} \tag{1}$$

Dependence of the air pressure p in the muscle on the muscle volume V and the volume flow rate of the compressed air to/from the muscle is given by differential equation:

$$\dot{p} + \frac{p}{V}\left(\frac{\partial V}{\partial h} + \frac{\partial V}{\partial d_2}\frac{\partial d_2}{\partial h}\right)\frac{\partial h}{\partial t} = \frac{R_S \cdot T}{V}\left(Q_{in} - Q_{out}\right) \tag{2}$$

where Q_{in}/Q_{out} is the air flow rate through the inlet/outlet valve, R_S is the specific gas constant of the air and T is the absolute temperature.

On the basis of the law of energy conservation the input virtual work $\mathrm{d}W_{in} = p \cdot \mathrm{d}V$ and the output virtual work $\mathrm{d}W_{out} = -F \cdot \mathrm{d}h$ done by muscle must be the same. Then dependence of the muscle tensile force F on the air pressure in the muscle can be obtained as follows:

$$F = -p \cdot \left(\frac{\pi \cdot d_1^2}{20} + \frac{8L^2 - 6h^2}{15\pi \cdot N^2} + \frac{d_1\sqrt{4L^2 - h^2}}{15N} - \frac{d_1 h^2}{15N\sqrt{4L^2 - h^2}}\right) \tag{3}$$

Acquired knowledge will be used in future work to create a dynamic simulation model in Matlab Simulink environment in order to simulate the dynamics of the manipulator movement with PAMs.

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EXTENDED ABSTRACT BOOK

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