19th International Summer School

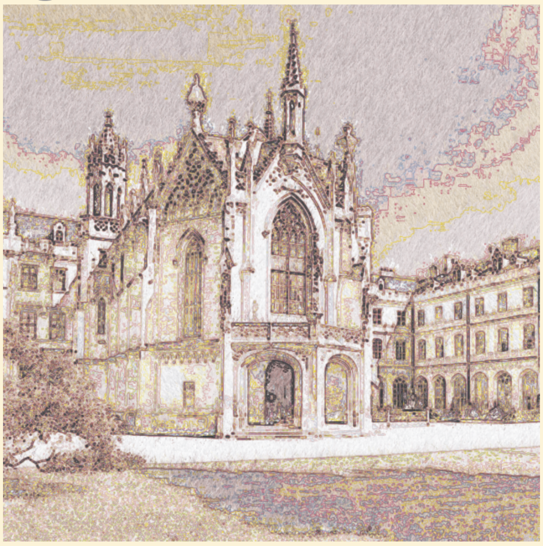
on

Global Analysis and its Applications

(EXTENDED ABSTRACT BOOK)

August 25 - 30, 2014

Lednice



Czech Republic

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19th International Summer School on Global Analysis and its Applications
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Scientific committee:
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Organizers:
Lepage Research Institute, Czech Republic

Department of Physics, Mathematics and Techniques, Faculty of Humanities and Natural Sciences, University of Prešov in Prešov, Slovakia

Editor:
Ján Brajerčík, University of Prešov in Prešov, Slovakia

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Preface

The 19th International Summer School on Global Analysis and its Applications has become a fruitful continuation of the series of summer schools focused on topics on the border of mathematics and physics, organized annually by the Lepage Institute Research group. "Symmetries" - the research program of this year meeting - were presented by renowned specialists in the field from different points of view: Professors Hernán Cendra (Universidad Nacional del Sur, Argentina), Peter Hydon (University of Surrey, United Kingdom) and Valentin Lychagin (University of Tromsø, Norway) gave a basic course with the following topics: Dirac Structures and their applications in physics (H. Cendra), Symmetry methods for differential and difference equations (P. Hydon), and Symmetries, invariants and factor-equations (V. Lychagin).

For the organizers, I wish to thank our speakers for their top-quality lecturing as well as discussions on different research topics, and also to all participants for their support and making the 2014 summer school in Lednice (Czech Rep.) very pleasant. We invite all of you to celebrate the next year anniversary summer school with the anniversary topic "General Relativity: 100 years after Hilbert". Last but not least, our thanks belong also to the main organizers: the Lepage Research Institute and the University of Prešov for their support.

Lednice, 30 August 2014 Zbyněk Urban

Chairman of the Organizing Committee

Programme

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Hernán Cendra

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Valentin Lychagin

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SOME METHODS OF DIFFERENTIAL SPACES IN SPACETIME GEOMETRY
Krzysztof Drachal

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İsmail Gök, Osman Ateş

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EXAMPLES OF APPLICATIONS OF DIRAC STRUCTURES

Hernán Cendra[1], **María R. Etchechoury**[2], **Sebastián J. Ferraro**[3]

[1]Departamento de Matemática, Universidad Nacional del Sur, Av. Alem 1253, 8000 Bahía Blanca, ARGENTINA
[2]Departamento de Matemática, Universidad Nacional De La Plata, Calle 50, y 115, La Plata, ARGENTINA
[3]Departamento de Matemática, Universidad Nacional del Sur, Av. Alem 1253, 8000 Bahía Blanca, ARGENTINA
E-mail address: hcendra@gmail.com, marila.mate@gmail.com, sferraro@uns.edu.ar

Abstract

Dirac geometry has evolved since the discovery of Dirac structures about 30 years ago by Courant and Weinstein, [6a], [6b], as a generalization encompassing and going beyond presymplectic and Poisson geometry. In this course we will emphasize the meaning of Dirac geometry for modelling physical systems rather than its connection with other branches of mathematics.

A Dirac structure $D \subseteq TM \oplus T^*M$ is a vector subbundle of the Pontryagin bundle of the manifold M having certain properties that are satisfied, in particular, when D is the graph of the flat operator associated to a presymplectic form or the sharp operator associated to a Poisson bivector. The transformation properties of Dirac structures make it easier to use a categorical approach, where the objects are Dirac manifolds (M, D) and there are forward and backward morphisms, [3], [2]. This confers some advantages to Dirac geometry over Poisson geometry or symplectic geometry since certain transformations of Dirac structures may be well defined in certain situations where, for instance, Poisson geometry cannot be applied. For example, the pullback of a Poisson structure under a smooth map cannot be defined in general as a Poisson structure, but in many cases of interest the backward morphism of its graph is a Dirac structure.

In this course, we will describe essentially two aspects of Dirac geometry, both related to physical systems modelling.

In the first place, Dirac structures provide a way of describing equations of motion as Dirac systems of the type $(x, \dot{x}) \oplus d\mathcal{E} \in D$, where $\mathcal{E}$ is an energy function, [5], [14a], [14b]. This approach is useful for instance in representing Euler-Lagrange equations for singular Lagrangians, where the Dirac theory of constraints, [9], should be used to obtain a Hamiltonian description. The latter is related to the original motivation in [6a] for introducing Dirac structures. Examples where, besides the constraints coming from the singularities of the Lagrangian, there are "external" constraints like nonholonomic systems or LC circuits, control systems, etc, can also be represented as Dirac systems, which provides a unified formalism for all of them.

We will start with a review of the Dirac theory of constraints [9] and the Gotay-Nester theory, [10]. Constraints theories have been studied extensively by many authors, for instance, [4a], [4b], [12a], [12b]. In an abstract setting, the basic ingredients of the Dirac theory are a given symplectic manifold (a cotangent bundle in the original Dirac's exposition), the primary constraint submanifold (the image of the Legendre transformation in the original Dirac's exposition) described regularly as the zero set of a family of functions (primary constraints) and an Energy function (Hamiltonian) defined on the symplectic manifold. A generalization is given in [5] by replacing the primary constraint submanifold by a foliated submanifold, in order to deal with more general Dirac systems rather than the systems that originally motivated the Dirac or Gotay-Nester theories. As a further example we will show how to deal with linear Poisson systems using a Dirac geometric approach, which gives the equations of motion derived in [8]. One should also mention that, in the context of Dirac structures, it appeared recently a study of constraints in which the symplectic manifold (phase space) is replaced by a Poisson manifold, [3].

Second, the other important aspect is that Dirac geometry can be used to study systems with ports and interconnected systems [13]. This approach is specially useful in the case of complex systems. We will define the notions of port-Dirac structures and systems (which are closely related to port-Hamiltonian systems, [13]) and we will briefly show how the categorical approach helps to understand this notion and also the notion of interconnection.

We will very briefly review the notion of Dirac-Weinstein reduction (work in progress).

The list of references below is by no means complete.

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SYMMETRY METHODS FOR DIFFERENTIAL AND DIFFERENCE EQUATIONS

Peter E Hydon

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF SURREY, GUILDFORD GU2 7XH UNITED KINGDOM

E-mail address: P.Hydon@surrey.ac.uk

ABSTRACT

Most well-known techniques for solving differential equations exploit symmetry in some form. This simple observation, due to Sophus Lie, has been developed into a set of systematic methods for finding and using symmetries, first integrals and conservation laws of a given differential equation. Recent research has shown how to extend these powerful methods to difference equations. For instance, there are difference analogues of Noether's two theorems on variational symmetries, together with a new intermediate result. An important application is to determine which finite difference approximations retain conservation laws, Bianchi identities and other essential structures.

This course describes the basic theory for both differential equations (DEs) and difference equations ($\triangle$Es), and shows how the resulting techniques are used in practice. The contents and conclusions of each lecture are summarized below, together with some suggestions for further reading.

Lecture 1: From symmetries to solutions

- Most well-known methods for solving a given first-order ODE or O$\triangle$E use canonical coordinates to transform the equation into a simple solvable form.
- Symmetries of a given ODE or O$\triangle$E map the set of solutions to itself (invertibly, preserving the locally-smooth dependence on arbitrary constants).
- Nontrivial one-parameter local Lie groups of symmetries yield useful local canonical coordinates.

Lecture 2: How to find Lie symmetries

- Given an ODE or O△E of order $p \geq 2$, canonical coordinates reduce the order by one. If the reduced equation can be solved, the original equation is solved by one more integration or summation.
- The infinitesimal generator X determines the local behaviour of Lie symmetries.
- A function is invariant if and only if it is in the kernel of X.
- The prolonged infinitesimal generator yields the linearized symmetry condition (LSC). For an ODE, this is a differential equation; for an O△E, the LSC is a functional equation.
- By restricting the characteristic, Q, one can seek all Lie symmetries that satisfy the restriction.
- Point symmetries act on the space of independent and dependent variables; dynamical symmetries act on the space of first integrals (or arbitrary constants).

Lecture 3: Lie symmetry methods for PDEs and P△Es

- Provided that the LSC can be written as an identity, one can find its Lie symmetries by the same methods as for ODEs or O△Es.
- Besides Lie point symmetries, PDEs and P△Es may also have generalized symmetries.
- Invariant solutions, which satisfy the given equation and $Q = 0$, may be constructed from Lie point or generalized symmetries.
- Lie point symmetries can be used to construct linearizing point transformations for a given PDE or P△E, provided such transformations exist.

Lecture 4: Conservation laws

- For PDEs, characteristics identify equivalent conservation laws directly.
- Characteristics for P△E conservation laws can be constructed; their roots identify equivalent conservation laws.

- Characteristic discretization of continuous characteristics can produce finite difference schemes that preserve multiple conservation laws.

Lecture 5: Noether-type theorems

- Noether's theorems on variational symmetries apply equally to PDEs and P$\triangle$Es.
- There is a result that bridges the gap between Noether's theorems.
- For at least some gauge theories, there exist discretizations that preserve the relations produced by Noether's Second Theorem.
- A key reason for the close analogy between PDE and P$\triangle$E methods is the existence of identical cohomological structures.

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SYMMETRIES, INVARIANTS AND FACTOR-EQUATIONS

Valentin Lychagin

Department of Mathematics, University of Tromsø, NORWAY
E-mail address: valentin.lychagin@uit.no

Abstract

The main purpose of these lectures is to present a short introduction to the world of differential invariants. We'll discuss symmetry groups and pseudogroups of PDEs and algebras of their differential invariants. The main theorem in the differential invariants theory - Lie-Tresse theorem - shall be discussed in details. We shall explain in details the importance of certain kind of algebraicity in PDEs theory and outline algebro-differential methods for finding differential syzygies and construction of the factor equations. A number of concrete examples shall be considered.

SOME METHODS OF DIFFERENTIAL SPACES IN SPACETIME GEOMETRY

Krzysztof Drachal

Faculty of Mathematics and Information Science, Warsaw University of Technology, Koszykowa 75, 00 – 662 Warszawa, Poland

E-mail address: k.drachal@mini.pw.edu.pl

Abstract

Let M be a nonempty set. Let $\mathcal{A}$ be a family of real functions on M. Denote these functions by f_i. Let this family fulfill axioms of the Sikorski differential space, i.e. let the pair $(M, \mathcal{A}_M)$ be the Sikorski differential space.

Let a predifferential space be the construction similar to Sikorski one, but without the axiom of localization closure.

Let $\widehat{\mathcal{A}}$ denote the collection of real functions defined on $\mathrm{Spec}\mathcal{A}$ by the following rule $\widehat{a}(\chi) = \chi(a)$, where $a \in \mathcal{A}$, $\widehat{a} \in \widehat{\mathcal{A}}$ and $\chi \in \mathrm{Spec}\mathcal{A}$.

Proposition 1 *For the predifferential space $(M, \mathcal{A})$ the following statements are equivalent:*

- $(M, \mathcal{A})$ *has the spectral property.*
- *The generator image $F(M)$ is closed with respect to the Euclidean metric.*
- $(M, \mathcal{A})$ *and* $(\mathrm{Spec}\mathcal{A}, \widehat{\mathcal{A}})$ *are diffeomorphic.*

Let (M, g) be a spacetime, i.e. M is 4–dimensional, smooth manifold and g is Lorentzian metric. Let $O(M)$ be the connected component of the fibre bundle of orthonormal frames over M.

Let $\overline{\mathcal{A}}^{\mathrm{inv}}$ denote $O(3,1)$–invariant functions from $C^\infty(\overline{O(M)})$ and let $\mathcal{A}^{\mathrm{inv}}_{O(M)}$ denote $O(3,1)$–invariant functions from $\mathcal{A}_{O(M)}$.

Proposition 2 *The original Schmidt's b–boundary of a spacetime is then just*

$$(\mathrm{Spec}\overline{\mathcal{A}}^{\mathrm{inv}}, \widehat{\overline{\mathcal{A}}^{\mathrm{inv}}}) \setminus (\mathrm{Spec}\mathcal{A}^{\mathrm{inv}}_{O(M)}, \widehat{\mathcal{A}^{\mathrm{inv}}_{O(M)}}) \quad .$$

Now consider $E = O(M)$ and $G = O(3,1)$. Let $E \overset{\pi_M}{\to} M$ be the fiber bundle.

Space E is a sum of orbits of the right action $E \times G \to E$ of the group G on E, i.e. $E = \bigcup_{x \in M} \pi_M^{-1}(x)$.

Consider the Cauchy completion $\overline{E}$ of the space E (with respect to a fixed metric). The mentioned right action may be prolonged to $\overline{E} \times G \to \overline{E}$.

Consider two algebras of G–invariant functions on E and on $\overline{E}$, i.e.

$$\mathcal{F}_G(E) := \{f \in C^\infty(E) \mid \forall_{g \in G, p \in E} f(pg) = f(p)\}$$

and $\mathcal{F}_G(\overline{E}) := \{\overline{f} \mid f \in \mathcal{F}_G^0(E)\}$, where $\mathcal{F}_G^0(E)$ is subalgebra of $\mathcal{F}_G(E)$ consisting of functions which can be continuously prolonged on $\overline{E}$.

One can always choose the generators f_i of $C^\infty(E)$ such that they are prolongable, due to the Nash theorem.

The algebra of G–invariant functions from $C^\infty(\overline{E})$ is isomorphic to $\mathcal{F}_G(\overline{E})$.

Moreover $\mathcal{F}_G(E) = \pi_M^*(C^\infty(M)) = (\mathrm{sc}\{\pi_M^* h_1, \ldots, \pi_M^* h_n\})_E$, where h_i are generators of $(M, C^\infty(M))$.

Algebras $\mathcal{F}_G^0(E)$ and $\mathcal{F}_G(\overline{E})$ are isomorphic. So singularities of b–boundary can be classified basing on the relation between algebras $\mathcal{F}_G(E)$ and $\mathcal{F}_G(\overline{E})$ (equivalently: $\mathcal{F}_G^0(E)$ and $\mathcal{F}_G(E)$).

Proposition 3 *There are three cases:*

[1] $\mathcal{F}_G^0(E) = \mathcal{F}_G(E)$ *and then* $\mathrm{Spec}\mathcal{F}_G(E) = \mathrm{Spec}\mathcal{F}_G^0(E)$,

[2] $\mathbb{R} \ncong \mathcal{F}_G^0(E) \subsetneq \mathcal{F}_G(E)$,

[3] $\mathcal{F}_G^0(E) \cong \mathbb{R}$ *and then* $\mathrm{Spec}\mathcal{F}_G^0(E) = \{*\}$.

($\mathbb{R}$ above represents constant functions.)

In the first case the boundary is empty.

In Case 2 there are two subcases:

2a. $(\mathcal{F}_G^0(E))_E = \mathcal{F}_G(E)$ and then the boundary corresponds to $\overline{M} \backslash M$,

2b. $\mathcal{F}_G^0(E) \subsetneq \mathrm{sc}\{\pi_M^* h_1, \ldots, \pi_M^* h_n\}$.

Case 2b is possible only if some nonprolongability emerges at the level of the generators $h_1, \ldots, h_n$ of M. This can happen if the topology on $\overline{O(M)}$ is stronger than the topology induced by π.

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MSC2010: 58A40, 83C75
Key words: B-boundary, differential spaces, spacetime singularities.

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NOTES ON SLANT CURVES IN SASAKIAN 3-MANIFOLDS

İsmail Gök, Osman Ateş

Department of Mathematics, Faculty of Science, Ankara University 06100, Ankara - TURKEY

E-mail address: igok@science.ankara.edu.tr, ateso@ankara.edu.tr

Abstract

In the study of curves theory in differential geometry, special curves such as geodesics, circles, Bertrand curves, circular helices, general helices, slant helices etc. play an important role. Characterizations of these curves have been studied for a long time. A curve of constant slope or helix is defined by the property that its tangent vector field makes a constant angle θ with a fixed line l which is axis of the curve in space. A necessary and sufficient condition that a curve be of constant slope or a general helix is that the ratio of curvature to torsion be constant. This classical result stated by M. A. Lancret in 1802 [9] and first proved by B. de Saint Venant in 1845 [4].

Izumiya and Takeuchi [10] have introduced the concept of slant helices and conical geodesic curves in Euclidean 3-space. A slant helix in Euclidean space E^3 was defined by the property that its principal normal vector field makes a constant angle with a fixed line u. Moreover, they give a classification of special developable surfaces under the condition of the existence of such a special curve as a geodesic.

As a generalization of Legendre curves, Cho et.al [6] have introduced the notion of a slant curve. In their study, a curve in a contact manifold is said to be slant helix if its tangent vector field makes a constant angle with the Reeb vector field ξ. In particular, if the contact angle is equal to $(\pi/2)$, then the curve is called a Legendre curve. Morever, in another study, Cho et.al [7] have given that biharmonic curves in 3-dimensional Sasakian space forms are slant helices. After these studies, Ji-Eun Lee and et.al [5] introduced the notion of C-parallel mean curvature vector fields and C-proper mean curvature vector fields along slant helices in Sasakian 3-manifolds.

In this study, firstly we give some characterizations of slant curve and generalized it to new concept of curve called N -slant curve whose principal normal vector field makes a constant contact angle with the Reeb vector field ξ. Moreover, we introduce the notion of AW(k) types of slant helices and obtain mean curvature vector fields along the curves in Sasakian 3-manifolds.

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MSC2010: 53C25, 53B25
Keywords: slant helices, Sasakian manifolds.

EXISTENCE AND PERIODICITY FOR SOLUTIONS OF DELAY DIFFERENTIAL EQUATIONS

Ali Khelil Kamel, Ahcene Djoudi

Department of Mathematics, Badji Mokhtar University, Annaba, ALGERIA
E-mail address: k.alikhelil@yahoo.fr, adjoudi@yahoo.com

Abstract

In this work, we use the Krasnoselskii fixed point theorem to show that some kind of neutral differential equation with delay has a periodic solution. Also, by transforming the problem to an integral equation we are able, using the contraction mapping principle, to show that the periodic solution is unique.

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MSC2010: 47H10
Keywords: neutral differential equation, Krasnoselskii theorem, periodic solution, nonlinear differential equation, delay differential equation.

TOPOLOGY, RIGID COSYMMETRIES AND LINEARIZATION INSTABILITY IN HIGHER GAUGE THEORIES

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Igor Khavkine

Department of Mathematics, University of Trento, Via Sommarive, 14, Povo (TN) 38123, ITALY

E-mail address: igor.khavkine@unitn.it

Abstract

It is well known that the (in general infinite dimensional) solution spaces of non-linear PDEs may possess algebraic singularities, that is, singularities similar to those of finite dimensional algebraic varieties. Practically, not all solutions of the linearized PDE about such a singular point (constituting its *formal* or *Zariski* tangent space) belong to the set of those that are tangent to a 1-parameter family of exact solutions (constituting the *tangent cone*). This inequality between the Zariski tangent space and the tangent cone are referred to as *linearization instability*.

Seminal work in the '70s on equations of mathematical physics by Fischer, Marsden, Moncrief and others [2, 4] established that linearization instability occurs in General Relativity, Yang-Mills and other gauge theories, and that they are linked to compact spatial topology and Killing-type symmetries. Unfortunately, that work has been mostly of case-by-case type and in some ways non-geometric, relying significantly on functional analytical methods. My work, on the other hand, introduces a systematic way and fully geometric way of discovering sufficient conditions for linearization instability. All known examples happen to be of that form and some new examples can be readily identified. These results partially overlap with the little known work of Arms & Anderson [1].

The basic idea, given an exact solution φ of a non-linear PDE $e[\varphi] = 0$, is to look for *linearization obstructions*, that is functions $Q(\psi)$ on solutions of the linearized PDE $e_\varphi[\psi] = 0$ such that $Q(\psi) \neq 0$ implies that ψ is not the linearization of a 1-parameter family of exact solutions passing through φ. Such

obstructions can be built from bilinear pairings of conservation laws of e_φ and de Rham cohomology classes of the domain manifold.

The key construction is of the *compatibility* (also *Noether*) *complex* for the linear operator e_φ. Namely we call z^0_φ a *compatibility operator* for e_φ if $z^0 \circ e_\varphi = 0$ and any other differential operator g such that $g \circ e_\varphi = 0$ factors through z^0_φ. We inductively define z^{i+1}_φ to be the compatibility operator of z^i_φ, and for convenience set $z^{-1}_\varphi = e_\varphi$. The cohomology classes in $H^i_{\mathrm{def}}(e_\varphi) = \ker z^i_\varphi / \operatorname{im} z^{i-1}_\varphi$ are called *stage-i consistent deformations*. Note that the leading non-linearity $f_\varphi[t\psi] = e[\varphi + t\psi] - te_\varphi[\psi] + \cdots$ represents an element in $H^0_{\mathrm{def}}(e_\varphi)$.

The formal adjoints z^{i*}_φ constitute the *adjoint compatibility complex*, since the property $z^{(i-1)*}_\varphi \circ z^{i*}_\varphi$ is preserved. For variational PDEs, the operator z^{0*}_φ coincides with the generator of infinitesimal gauge symmetries, with *higher gauge theories* characterized by the non-triviality of z^{i*}_φ $(i > 0)$. The cohomology classes in $H^i_{\mathrm{cosym}}(e_\varphi) = \ker z^{(i-1)*}_\varphi / \operatorname{im} z^{i*}_\varphi$ are called *rigid cosymmetries*. A generalization of Noether's first theorem [3] establishes an isomorphism between $H^i_{\mathrm{cosym}}(e_\varphi)$ and $H^i_{\mathrm{char}}(e_\varphi)$, which consists of equivalence classes degree i-differential forms (both cosymmetries and these forms may depend locally on ψ and its derivatives) that are closed modulo the equations $e_\varphi[\psi] = 0$. Traditionally, the representatives of $H^i_{\mathrm{char}}(e_\varphi)$ are called *higher conservation laws*.

A special construction pairs the leading non-linearity $[f_\varphi]$ (or any other element in $H^0_{\mathrm{def}}(e_\varphi)$) with a $[\rho] \in H^i_{\mathrm{cosym}}(e_\varphi)$ to produce a *deformation current* $[j] \in H^{n-i}_{\mathrm{char}}(e_\varphi)$, with n being the dimension of the domain manifold M. We can think of the de Rham cohomology valued $Q(\psi) = [j[\psi]] \in H^{n-i}_{\mathrm{dR}}(M)$ as the charge of $\psi \in T_\varphi$ with respect to this current, where T_φ denotes the space of linearized solutions. The main result of this work is that the charge $Q(\psi)$ must vanish on linearized solutions that belong to the tangent cone at φ. In other words, any i and l define a linearization obstruction map

$$Q^l_i \colon T_\varphi \to {}^{(l)}H^i_{\mathrm{cosym}}(e_\varphi)^* \otimes H^{n-i}_{\mathrm{dR}}(M),$$

where ${}^{(l)}H^i_{\mathrm{cosym}}(e_\varphi)^*$ denotes the linear dual of the subspace of rigid cosymmetries that are homogeneous polynomials in ψ and its derivatives.

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Keywords: nonlinear PDEs, linearization instability, compatibility complex, Noether complex, higher gauge theory, symmetry, cosymmetry, de Rham cohomology.

THE NOETHER'S THEOREMS AND NATURAL VARIATIONAL PRINCIPLES

Demeter Krupka

Lepage Research Institute, CZECH REPUBLIC
E-mail address: demeter.krupka@lepageri.eu

Abstract

The aim of this note is to discuss basic geometric aspects of the theory of invariant variational principles. Let Y be a smooth manifold, α a diffeomorphism of Y. We say that a p-form ρ on Y is *invariant with respect to* α, if its pull-back $\alpha^*\rho$ coincides with α, that is, $\alpha^*\rho = \rho$. This definition immediately extends to vector fields.

If ξ is a vector field on Y, then ξ generates invariance transformations of ρ if and only if the Lie derivative of ρ by ξ vanishes, $\partial_\xi \rho = 0$. Since the well known (Cartan's) formula $\partial_\xi \rho = i_\xi d\rho + di_\xi \rho$ is valid, for such a vector field we have $i_\xi d\rho + di_\xi \rho = 0$. In the lecture we show that for some special choices of Y, ρ and ξ, this formula reduces to the Noether's equation known in the calculus of variations; the forms $i_\xi d\rho$ and $i_\xi \rho$ reduce to the *Euler-Lagrange form* and the *conserved current*. If in addition a mapping of manifolds $\gamma : X \to Y$ satisfies $\gamma^* i_\xi d\rho = 0$, then $d\gamma^* i_\xi \rho = 0$ (first Noether's theorem).

Let τ be a covariant functor from the category $\mathcal{D}_n$ of smooth n-dimensional manifolds and their diffeomorphisms to the category $\mathcal{FB}_n$ of associated bundles over n-dimensional manifolds and their isomorphisms. Then for every object $Y \in \mathrm{Ob}\mathcal{FB}_n$ over an object $X \in \mathrm{Ob}\mathcal{D}_n$, Y can be written as $Y = \tau X$; every morphism $\alpha \in \mathrm{Mor}\mathcal{D}_n$, $\alpha : X \to X$, induces a morphism $\tau\alpha : \tau X \to \tau X$. An n-form ρ on τX is said to be *natural*, if $(\tau\alpha)^*\rho = \rho$ for all α. Similar construction using flows applies to the vector fields ξ on X. Then the naturality implies $i_{\tau\xi} d\rho + di_{\tau\xi}\rho = 0$ for all vector fields on X. Thus, if $\gamma : X \to \tau X$ is a section satisfying the equations $\gamma^* i_\xi d\rho = 0$, we get $d\gamma^* i_{\tau\xi}\rho = 0$ for all ξ (second Noether's theorem). We show that for some special choices of τ and ρ, this formula reduces to the second *Noether's* theorem in the calculus of variations on natural fibre bundles.

MSC2010: 49S05, 58A32
Keywords: variational principle, invariance, Noether's theorem, Euler-Lagrange form, conservation law.

SEMIHOLONOMIC VELOCITIES AND CONTACT ELEMENTS: AN ALGEBRAIC APPROACH I, II

Miroslav Kureš

Institute of Mathematics, University of Technology, Technická 2, 61669 Brno, CZECH REPUBLIC

E-mail address: kures@fme.vutbr.cz

Abstract

In the first part of the workshop, we explain the basic concepts and results related to Weil bundles. We will focus on nonholonomic and semiholonomic velocities and the corresponding Weil algebras. In the second part, we will study Weil contact elements. The whole workshop will be interspersed with references to geometric problems, in particular to liftings of geometric objects, and possible applications in physics will be mentioned, too.

Selected author's papers related to the topic from the most recent period:

Kureš, M., Weil algebras associated to functors of third order semiholonomic velocities, Mathematical Journal of Okayama University, Vol. 56, (2014), No. 1, pp. 117-127, ISSN 0030-1566, Okayama University.

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MSC2010: 58A32, 58A20

Keywords: Weil algebra, nonholonomic jet, semiholonomic jet, higher order velocity.

PRINCIPAL FUNCTIONS OF NON-SELFADJOINT MATRIX STURM-LIOUVILLE EQUATIONS

Murat Olgun, Yelda Aygar

Department of Mathematics, Faculty of Science, University of Ankara 06100 Ankara-TURKEY

E-mail address: olgun@ankara.edu.tr, yaygar@science.ankara.edu.tr

Abstract

Consider the boundary value problem (BVP)

$$-u'' + q(x)u = \lambda^2 u, \quad x \in \mathbb{R}_+ \tag{1}$$

$$u(0) = 0, \tag{2}$$

in $L^2(\mathbb{R}_+, E)$, where q is a complex-valued function. The spectral theory of the above BVP with continuous and point spectrum was investigated by Naimark [5]. He showed the existence of the spectral singularities in the continuous spectrum of the BVP. He noted that the eigenfunctions and the associated functions (principal functions) corresponding to the spectral singularities are not the elements of $L^2(\mathbb{R}_+)$. He also noted that the spectral singularities belonging to the continuous spectrum are the poles of the resolvent's kernel, but are not the eigenvalues of the BVP. Their existence does not occur in the spectral theory of selfadjoint BVP. Spectral analysis of the selfadjoint differential and difference equations with matrix coeficients has been studied by the works shown in [3, 6, 7].

Let E be an n-dimensional ($n < \infty$) Euclidian space and let the Hilbert space of vector-valued functions with the values in E be denoted by $L^2(\mathbb{R}_+, E)$. In the $L^2(\mathbb{R}_+, E)$ space we consider the BVP

$$-y'' + Q(x)y = \lambda^2 y, \quad x \in \mathbb{R}_+ \tag{3}$$

$$y(0) = 0, \tag{4}$$

where Q is a non-selfadjoint matrix-valued function (i.e. $Q \neq Q^*$). It is clear that, the BVP (3), (4) is non-selfadjoint. In this work, we aim to investigate

the properties of the principal functions corresponding to the eigenvalues and the spectral singularities of the BVP shown in equations (3), (4).

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MSC2010: 34B24,34L40, 47A10
Keywords: eigenvalues, spectral singularities, spectral analysis, Sturm-Liouville operator, non-selfadjoint matrix operator.

NONLINEAR STATIC CHARACTERISTIC OF PNEUMATIC MUSCLE ACTUATOR

Ján Piteľ, Mária Tóthová

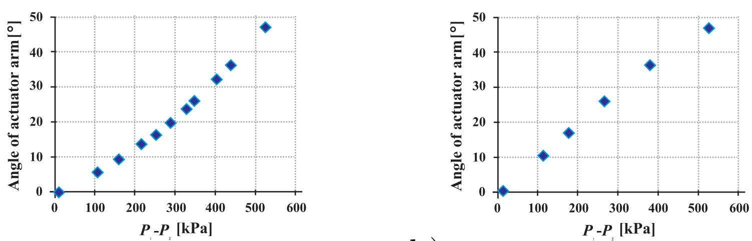
Department of Mathematics, Informatics and Cybernetics, Faculty of Manufacturing Technologies, Technical University of Košice, SLOVAKIA

E-mail address: jan.pitel@tuke.sk, maria.tothova@tuke.sk

Abstract

Pneumatic muscle actuator (PMA) consisting of a pair of artificial muscles in antagonistic connection belongs to nonconventional actuators with some properties similar to biological muscles. Arm position of such actuator depends mainly on pressure difference in the muscles.

The static characteristic of the actuator was measured on experimental PMA with two artificial muscles type MAS 20-250N FESTO in antagonistic connection. Input variable is pressure difference in the muscles $(P_1 - P_2)$ and the output variable is the angle φ of actuator arm rotation. Due to the complex elastic properties of the muscles as well as the compressibility of the air, this characteristic is nonlinear. At the beginning both muscles were fully pressurized and actuator arm was in zero initial position ($\varphi = 0°$). Measured results are shown in Fig. 1, part a) is for the air discharge from the muscle and part b) is for the air filling into the muscle.



a) b)

Fig.1 Measured results of dependence of the angle of actuator arm rotation on pressure difference in the muscles

The approximation of these measured results was made using the method of least squares in MS Excel. Regarding of shape of the measured characteristic the initial form of the approximating function was found as polynomial function:

$$\varphi = f(P, A) = f(P_1 - P_2, a_0, a_1, \ldots, a_k), \tag{1}$$

where the parameters a_i, $i = 1, \ldots, k$ must be determined so that the approximated curve to be close to the measured points. This requirement will be fulfilled if we consider the minimum of function:

$$S(A) = \sum_{j=1}^{n} [\varphi_j - f(P, A)]^2. \tag{2}$$

Nonlinear function for the air discharge from the muscle is expressed by the third degree polynomial (with approximation error $AE = 0.9997$):

$$y = 8 \cdot 10^{-9} x^3 + 8 \cdot 10^{-5} x^2 + 0.047x - 0.376. \tag{3}$$

Nonlinear function for the air filling into the muscle is expressed by the fourth degree polynomial (with approximation error $AE = 1$):

$$y = -9 \cdot 10^{-11} x^4 - 2 \cdot 10^{-8} x^3 + 3 \cdot 10^{-5} x^2 + 0.095x - 0.421. \tag{4}$$

Due to the fact that the static characteristic of the actuator is centrally symmetric function (Fig. 2), then based on (3) and (4) for the angle of the actuator arm rotation applies:

$$\begin{aligned} \varphi \;&=\; sign(P_1 - P_2) \cdot (8 \cdot 10^{-9} \, |P_1 - P_2|^3 + 8 \cdot 10^{-5} \, |P_1 - P_2|^2 + \\ &\quad +0.047 \, |P_1 - P_2| - 0.376), \end{aligned} \tag{5}$$

$$\begin{aligned} \varphi \;&=\; sign(P_1 - P_2) \cdot (-9 \cdot 10^{-11} \, |P_1 - P_2|^4 - 2 \cdot 10^{-8} \, |P_1 - P_2|^3 + \\ &\quad +3 \cdot 10^{-5} \, |P_1 - P_2|^2 + 0.095 \, |P_1 - P_2| - 0.421). \end{aligned} \tag{6}$$

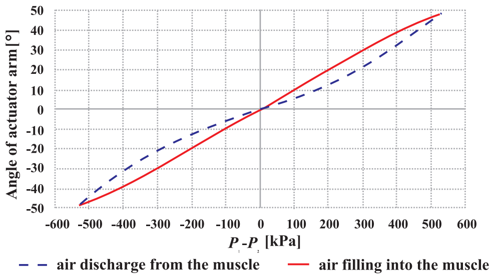


Fig.2 Static characteristic of PMA

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MSC2010: 70B15
Keywords: pneumatic muscle actuator, artificial muscle, static characteristic.

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UNIFORM PROJECTILE MOTION AS A NONHOLONOMIC SYSTEM WITH A NONLINEAR CONSTRAINT

Martin Swaczyna[1], **Petr Volný**[2]

[1]Department of Mathematics, Faculty of Science, University of Ostrava, 30. dubna 22, 701 03, Ostrava, CZECH REPUBLIC
[2]Department of Mathematics and Descriptive Geometry, VŠB - TU Ostrava, 17. listopadu 15, 708 33, Ostrava, CZECH REPUBLIC
E-mail address: martin.swaczyna@osu.cz, petr.volny@vsb.cz

Abstract

A uniform projectile motion (motion of a projectile with constant instantaneous speed) is an illustration of a behavior of a simple mechanical system of one particle subject to one nonlinear nonholonomic constraint. For a complete description of dynamics of the uniform projectile motion it is necessary to understand a requirement of constant instantaneous speed as a nonholonomic, called *isotachytonic* constraint. Since isotachytonic constraint represents general nonintegrable constraint, it is necessary to adopt a modern approach based on the geometric concept of nonholonomic mechanical systems.

A geometric treatment of the problem of uniform projectile motion as a nonholonomic system of one particle with a nonlinear constraint is presented. We apply a general geometric theory of nonholonomic mechanical systems. The problem is investigated from the kinematic and dynamic point of view. Corresponding kinematic parameters of classical and uniform projectile motion are compared, nonholonomic Hamilton equations are derived and their solvability is discussed. Finally, symmetries and conservation laws of the uniform projectile motion are studied, the nonholonomic formulation of a conservation generalized law of energy is found as one of the corresponding Noetherian first integrals of the nonholonomic system.

This contribution is based on the paper [1] which contains the following main results:

- The explicit expressions of standard kinematic parameters (range, height and total time) of uniform projectile motion are derived.
- Additional parameters (distance deviation and directional deviation) comparing isochronic points of both trajectories are introduced and its time behavior during the motions is graphically illustrated.
- Realization of uniform projectile motion by a regulation of some external force acting along the line of instantaneous velocity is proposed.
- Hamilton approach to the uniform projectile motion problem is presented: constraint momenta and constraint energy 1-form are computed and corresponding nonholonomic Hamilton equations are obtained.
- The set of all constraint Noetherian symmetries of the system was found and physical interpretation of corresponding conservation laws is presented.
- One of the obtained Noetherian first integral is interpreted as a generalized conservation law of energy.
- Energetic balances of classical and uniform projectile motions are compared.
- Conservation laws of energy of both motions are presented in a unified form, in the case of uniform projectile motion the law moreover includes some factor called *isotachytonic compensation coefficient*.
- Relation between generalized conservation law of energy and the mechanical work of Chetaev constraint force is discussed.

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MSC2010: 37J60, 70F25, 70G45, 70H03, 70H05, 70H33
Keywords: nonholonomic mechanical systems, constrained Noetherian symmetries, nonholonomic conservation laws.

INVARIANT LEPAGE FORMS ON GRASSMANN FIBRATIONS: EXAMPLES

Zbyněk Urban, Demeter Krupka

Lepage Research Institute, CZECH REPUBLIC

E-mail address: zbynek.urban@lepageri.eu, demeter.krupka@lepageri.eu

Abstract

Our aim is to characterize the meaning of the definition of the first order Lepage form, the *Hilbert form*, its invariance properties and the consequences arising from the Noether theorem in this context ("conservation laws") for extremals of the underlying variational functionals. In particular, differences between basic underlying concepts, the *Lagrange form*, *Euler-Lagrange form*, etc. on fibred manifolds on one side, and the *Lagrange class*, *Euler-Lagrange class*, etc. on *Grassmann fibrations* are described. The methods, based on the Hilbert form, do not require any parametrisations. The examples also provide us with the methods how the invariance properties can be used for the study of extremals: to this purpose one should first solve the Noether equations for the generators of invariance groups, and then to formulate and solve the "conservation law" equations. It turns out that in our examples the *conservation law equations* are completely equivalent with the Euler-Lagrange equations for extremals. The functions, representing the first integrals, or "conserved quantities", can naturally be interpreted as a part of the *adapted coordinates* to the extremal submanifolds. We consider in our examples three variational functionals for 1-dimensional (nonparametrized) submanifolds of $\mathbb{R}^2$ and $\mathbb{R}^3$, defined on the Grassmann fibrations $G^1\mathbb{R}^2$ and $G^1\mathbb{R}^3$; the functionals are defined by means of the 1st-order Lepage forms (the Hilbert forms). The following topics are included:

- construction of the Hilbert form from a homogeneous Lagrangian,
- the Euler-Lagrange class and equations for extremals as set solutions,
- invariance transformations of the Lagrange class, classification,
- "conserved quantities" (first integrals) within the Grassmann fibration framework,
- conservation law equations and their solutions.

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MSC2010: 58E30, 58D19, 53C40, 58A20.

Keywords: Grassmann fibration, Lagrangian, Lepage form, Hilbert form, invariance, Noether current.

REPRESENTATION OF VARIATIONAL SEQUENCES

Jana Volná

VŠB-Technical University of Ostrava, 17. listopadu 15, 708 33 Ostrava-Poruba, CZECH REPUBLIC

E-mail address: jana.volna@vsb.cz

Abstract

In this extended abstract a brief recapitulation of a representation of variational sequences is presented. Some additional properties are mentioned. A more detailed description can be found for example in [4] (mechanics) or in [6,7] (field theory).

Let $\pi : Y \to X$ be a fibred manifold with fibred coordinate systems (V, ψ), $\psi = (x^i, y^\sigma)$, on Y and (U, φ), $\varphi = (x^i)$ on X, $\dim X = n$, $\dim Y = m + n$. Denote by $\pi^r : J^rY \to X$ or just J^rY the r-jet prolongation of the fibred manifold $\pi : Y \to X$, the coordinate system is (V^r, ψ^r), $\psi^r = (x^i, y^\sigma, y^\sigma_j, y^\sigma_{j_1 j_2} \dots, y^\sigma_{j_1 j_2 \dots j_r})$ on J^rY. The canonical jet projections are $\pi^{r,s} : J^rY \to J^sY$, $r > s$ and $\pi^{r,0} : J^rY \to Y$.

A differential q-form ρ on J^rY is called *contact*, if it vanishes along the r-jet prolongation $J^r\gamma$ of every section γ of π. We say that an $(n + k)$-form ρ on J^rY is *strongly contact* if for every point $J^r_x\gamma \in J^rY$ there exist a fiber chart (V, ψ), $\psi = (x^i, y^\sigma)$, on Y, an integer $s \geq r$, and a contact $(n + k - 1)$-form η on V^s with $p_{k-1}\eta = 0$ such that

$$p_k((\pi^{s,r})^*\rho - d\eta) = 0. \tag{1}$$

For any open set $V \subset Y$ we denote by Ω^r_qV the Abelian group of differential forms defined on V^r and we denote by Θ^r_qV the Abelian group of contact forms $(q \leq n)$, resp. strongly contact forms $(q \geq n + 1)$ defined on V^r. Θ^r_qV is a subgroup of Abelian group Ω^r_qV. We have de Rham sequence

$$0 \to \mathbb{R}_Y \to \Omega^r_0 \to \Omega^r_1 \to \cdots \to \Omega^r_n \to \Omega^r_{n+1} \to \cdots \to \Omega^r_{N-1} \to \Omega^r_N \to 0, \tag{2}$$

of sheaves of Abelian groups on J^rY and its contact subsequence,

$$0 \to \Theta^r_1 \to \cdots \to \Theta^r_n \to \Theta^r_{n+1} \to \cdots \to \Theta^r_M \to 0, \tag{3}$$

in which all arrows denote the exterior differentiation d, and number $N = \dim J^rY$, number $M = m\binom{n+r-1}{n} + 2n - 1$. Both sequences are exact and the quotient sequence

$$\begin{aligned} 0 \to \mathbb{R}_Y \to \Omega_0^r \to \Omega_1^r/\Theta_1^r \to \dots \to \Omega_n^r/\Theta_n^r \to \dots \\ \dots \to \Omega_M^r/\Theta_M^r \to \Omega_{M+1}^r \to \dots \to \Omega_{N-1}^r \to \Omega_N^r \to 0 \end{aligned} \tag{4}$$

is also exact. We call the quotient sequence the *r-th order variational sequence.* The quotient mapping $E : \Omega_q^r/\Theta_q^r \to \Omega_{q+1}^r/\Theta_{q+1}^r$ is defined by $E([\rho]) = [d\rho]$. In the case $q = n$ we have the *Euler-Lagrange mapping* and in the case $q = n+1$ we have the *Helmholtz-Sonin mapping.*

For a representation of the variational sequence we use a variational projector $\mathcal{I}$, see [6,7]. At first we define auxiliary operators I_k, $k \geq 1$, acting on some special $(n+k)$-forms.

Let ρ be a 1-contact $(n+1)$-form on $J^{r+1}Y$, $\rho = \sum_{|J|=0}^{r} A_\sigma^J \omega_J^\sigma \wedge \omega_0$, then $I_1\rho$ is 1-contact ω^σ-generated $(n+1)$-form

$$I_1\rho = B_\sigma \omega^\sigma \wedge \omega_0, \qquad B_\sigma = \sum_{p=0}^{r} (-1)^p d_{i_1} \dots d_{i_p} A_\sigma^{i_1 \dots i_p}, \tag{5}$$

where $\omega^\sigma = dy^\sigma - y_i^\sigma dx^i$, $\omega_0 = dx^1 \wedge \dots \wedge dx^n$ and d_i is i-th total derivative.

Let $k > 1$, let ρ be a k-contact $(n+k)$-form on $J^{r+1}Y$, $\Xi_1, \Xi_2, \dots, \Xi_k$ be a π-vertical vector fields on Y and $\tilde{\Xi}_s = J^{2r+1}\Xi_s$ and $\hat{\Xi}_s = J^{r+1}\Xi_s$ be their prolongations. We define a k-contact $(n+k)$-form $I_k\rho$ on $J^{2r+1}Y$ by

$$\begin{aligned} i_{\tilde{\Xi}_k} \dots i_{\tilde{\Xi}_2} i_{\tilde{\Xi}_1} I_k\rho = \tfrac{1}{k} \Big(& i_{\tilde{\Xi}_k} \dots i_{\tilde{\Xi}_3} i_{\tilde{\Xi}_2} I_{k-1}(i_{\hat{\Xi}_1}\rho) - i_{\tilde{\Xi}_k} \dots i_{\tilde{\Xi}_3} i_{\tilde{\Xi}_1} I_{k-1}(i_{\hat{\Xi}_2}\rho) \\ & - \dots - i_{\tilde{\Xi}_k} i_{\tilde{\Xi}_1} i_{\tilde{\Xi}_{k-2}} \dots i_{\tilde{\Xi}_2} I_{k-1}(i_{\hat{\Xi}_{k-1}}\rho) - i_{\tilde{\Xi}_1} i_{\tilde{\Xi}_{k-1}} \dots i_{\tilde{\Xi}_2} I_{k-1}(i_{\hat{\Xi}_k}\rho) \Big). \end{aligned} \tag{6}$$

Let ρ be an arbitrary $(n+k)$-form on J^rY. Using the previous definition we set

$$\mathcal{I}\rho = I_k p_k \rho. \tag{7}$$

A $\mathbb{R}$-linear mapping $\mathcal{I} : \Omega_{n+k}^r \to \Omega_{n+k}^{2r+1}$, defined by (7), is called the *variational projector* (or interior Euler-Lagrange operator [7]). The form $\mathcal{I}\rho$ is called the *canonical representative* of ρ. For any open set W and for every form $\rho \in \Omega_{n+k}^r W$, $\mathcal{I}\rho$ belongs to the same class as $(\pi^{2r+1,r})^*\rho$. The kernel of the mapping $\mathcal{I}$ coincides with the Abelian group $\Theta_{n+k}^r W$, and $\mathcal{I}$ satisfies, up to the canonical jet projection, $\mathcal{I} \circ \mathcal{I} = \mathcal{I}$.

We can extend this definition also to the q-forms, where $q \leq n$. In this case we identify $\mathcal{I}$ with horizontalisation, i.e. $\mathcal{I}\rho = h\rho$ for q-forms where $q \leq n$.

Operator $\mathcal{I}$ represents a class of variational sequences by differential forms, does not depend on the choice of coordinates and is related to the concepts of calculus of variations (Lepage equivalent of Lagrangian, Euler-Lagrange form, Helmholtz form, see [1,2,3,5]). Taking the example of n-form, we can say that a differential n-form ρ is a Lepage equivalent of Lagrangian $\lambda = h\rho$ if and only if

$$\mathcal{I}d\rho = p_1 d\rho. \tag{8}$$

In this case the canonical representative of $d\rho$ (i.e. $\mathcal{I}d\rho$) is precisely Euler-Lagrange form of mentioned Lagrangian λ. This definition corresponds with known definition of Lepage equivalent of Lagrangians.

Let ε be a $\pi^{r,0}$-horizontal (source) form. Then the canonical representative of $d\varepsilon$ (i.e. differential form $\mathcal{I}d\varepsilon$) corresponds with known Helmholtz form with coefficients Helmholtz expressions. Then we can say that source form ε is variational if $\mathcal{I}d\varepsilon = 0$.

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NUMERICAL COMPUTATION AND PROPERTIES OF THE TWO DIMENSIONAL EXPONENTIAL INTEGRALS

Seyhmus Yardimci, İbrahim Erdal

Department of Mathematics, Faculty of Science, University of Ankara 06100 Ankara-TURKEY

E-mail address: yardimci@science.ankara.edu.tr, ibrahimerdal187@hotmail.com

Abstract

The two-dimensional exponential integral (TDEI) functions play an important role in various fields of theoretical physics, quantum chemistry, theory of transport process, theory of fluid flow and theory of radiative transfer in a multi-dimensional medium. The TDEI functions are especially useful for the study of anisotropic scattering in a two-dimensional medium with a scattering phase function. Breig and Crosbie derived a series expansion and recurrence relations suitable for numerical computation of the one-dimensional exponential integral functions. It is shown that the absorption of solar radiation by the earth's atmosphere is given in terms of first-order exponential integral function. The fundamental integral equation of the radiative transfer of two-dimensional planar media with anisotropic scattering was derived by Crosbie and Dougherty. Note that the TDEI functions are the kernel of that integral equation. The TDEI functions play an important role in the investigation of the two-dimensional radiative transfer in an absorbing-emitting cylindrical medium and determination of the radiative flux. The generalized exponential integral functions are studied. GEIF 's are expressed with the single integrals. In our study, GEIF 's are expressed with double improper integrals as given in the original expression. This depends on the truth that the uniform convergence of integrals gives more precise results. Also GEIF 's are given in terms of Bessel functions, in the form of series. This GEIF 's approximation gives ruder results compared to ours. This study uses a different methodology and results are achieved with higher accuracy.

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LIST OF PARTICIPANTS

Kamel Ali Khelil, Badji Mokhtar University, Algeria
Ján Brajerčík, University of Prešov, Slovakia, and Lepage Research Inst., Czech Republic
Hernán Cendra, Universidad Nacional del Sur, Argentina
Markus Dafinger, University Oldenburg, Germany
Milan Demko, University of Prešov, Slovakia, and Lepage Research Inst., Czech Republic
Krzysztof Drachal, Warsaw University of Technology, Poland
Jakub Dvorský, University of Ostrava, Czech Republic
İsmail Gök, Ankara University, Turkey
Peter Hydon, University of Surrey, United Kingdom
Igor Khavkine, University of Trento, Italy
Tetiana Klymchuk, Taras Shevchenko National University of Kyiv, Ukraine
Demeter Krupka, Lepage Research Institute, Czech Republic
Miroslav Kureš, University of Technology, Brno, Czech Republic
Valentin Lychagin, University of Tromsø, Norway
Radka Malíková, University of Ostrava, Czech Republic
Roman Matsyuk, Inst. Appl. Prbls. Mech. & Math., Ukraine
Murat Olgun, Ankara University, Turkey
Ján Piteľ, Technical University of Kosice, Slovakia
Martin Swaczyna, University of Ostrava, Czech Republic
Eriko Tanaka, Kagoshima University, Japan
Zbyněk Urban, Lepage Research Institute, and University of Pardubice, Czech Republic
Nico Voicu, Transilvania University, Romania, and Lepage Research Inst., Czech Republic
Jana Volná, VŠB - TU Ostrava, Czech Republic
Seyhmus Yardimci, Ankara University, Turkey

EXTENDED ABSTRACT BOOK

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