(b) Variational foundations of Finsler geometry: Projective spaces, Grassmann bundles and the Hilbert form (D. Krupka)

Contents

- preliminaries (review): projective spaces, projectivized tangent bundles, jets, regular jets, differential groups, jet manifolds, Grassmannians,
- Grassmann fibrations, prolongations, adapted coordinates
- differential forms and vector fields; Zermelo homogeneity condition
- lifting of immersions, variational functionals on Grassmann fibrations
- variations, Lepage forms: generalizations to Grassmann fibrations
- first variation formula, extremals
- Invariant variational principles on Grassmann fibrations
- Finsler variational functionals, Finsler metrics
- higher-order generalizations
- The Hilbert form
- geodesics, geodesic equations
- examples and applications
- the inverse problem of the calculus of variations for Finsler structures

Key references

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General higher order Grassmannians $G_{n,m}^r$ introduced (of order r and type (n,m), $n \leq m$); for $r = 1$, $n = 1$ (classical) projective space

D.R. Grigore, D. Krupka, Invariants of velocities and higher-order Grassmann bundles, J. Geom. Phys. 24 (1998) 244-264.

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S.S. Chern, W. Chen and K.S Lam, *Lectures on Differential Geometry*, World Scientific, Singapore, 1999

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Z. Urban, D. Krupka, Foundations of higher-order variational theory on Grassmann fibrations, Internat. J. Geom. Methods in Modern Physics 11 (2014) 1460023 (27 pages); DOI: 10.1142/S0219887814600238.

Global variational theory for 1-dimensional submanifolds, Lepage forms in this setting, complete basic theory

D. Krupka, Lepage forms in Kawaguchi spaces and the Hilbert form, Publ. Math. Debrecen 84 (2014), 147-164; DOI: 10.5486/PMD.2014.5791.

The Hilbert form - extension to higher order Lagrangians: fibered case