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Particle Motion and Continua in Relativity

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LAGRANGIAN APPROACH TO SPINNING OR RADIATING PARTICLE HIGHER-ORDER EQUATIONS OF MOTION IN SPECIAL RELATIVITY

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We recognize it worthwhile to distinguish among the higher-order equations of motion those admitting a variational reformulation. Considering Dixon's form of Papapetrou equations either for free particle,

$$p'=o, \quad S'=2p\wedge u, \tag{1}$$

or introducing an external force by means of $p'=ku'^2u$, and imposing within the restriction $\|u\|=1$ a supplementary condition $S'u+ku'=o$ one regains the Lorentz-Dirac equation, $mu'=k(u''+u'^2u)$, in the second case with constant mass $m=pu$, reversing thus the considerations of A.O.Barut. The Lorentz-Dirac equation, however, cannot be reformulated in terms of a variational principle*. In the presence of another supplementary condition, $Su=o$, the equations (1) can be given the form $mu'=Su''$ (M.Mathisson,1937), $u\wedge S'=o$ $(u^2=1)$. In terms of the star operator $*$, the spin four-vector $\|u\|s=\frac{1}{2}*(u\wedge S)$, and complying with the obvious constraint $su=o$, this can be rewritten as

$$mu'=*(u''\wedge u\wedge s) \quad (u^2=1,\ s'=o). \tag{2}$$

Renormalizing the mass by means of $m=m_*(1-(su)^2/s^2u^2)^{3/2}$, and taking m_* to be an arbitrary but fixed constant we state here that the following third-order equation of motion defining the same set of world lines as (2),

$$m_* \frac{u^2u'-(u'u)u}{\|s\|^3\|u\|^3} = \frac{*(u''\wedge u\wedge s)}{\|s\wedge u\|^3} - 3\,*(u'\wedge u\wedge s)\frac{(s\wedge u')\cdot(s\wedge u)}{\|s\wedge u\|^5},$$

is the Euler-Poisson equation of a parameter-invariant variational problem. Proceeding further in eliminating the spin variables from (2) results in the fourth-order equation $u'''=(p^2/s^2)u'$, $(u^2=1)$, which under the assumption $p^2>o$ was suggested from the higher-order lagrangian point of view by F.Riewe without indicating any direct relationship with Papapetrou equations.

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